4. A uniformly magnetized and conducting sphere of radius $R$ and total magnetic moment $m = 4\pi MR^3/3$ rotates about its magnetization axis with angular speed $\omega$. In the steady state, no current flows in the conductor. The motion is nonrelativistic; the sphere has no excess charge on it.

a) By considering Ohm’s law in the moving conductor, show that the motion induces an electric field and a uniform volume charge density in the conductor, \( \rho = -m\omega/(\pi c^2 R^3) \).

b) Because the sphere is electrically neutral, there is no monopole electric field outside. Use symmetry arguments to show that the lowest possible electric field multipolarity is quadrupole. Show that only a quadrupole field exists outside, and that the quadrupole moment has non-vanishing components \( Q_{33} = -4m\omega R^2/(3c^2), Q_{11} = Q_{22} = -Q_{33}/2 \).

c) By considering the radial electric fields inside and outside the sphere, show that the surface charge density \( \sigma(\theta) \) is

\[
\sigma(\theta) = \frac{1}{4\pi R^2} \frac{4m\omega}{3c^2} \left[ 1 - \frac{5}{2} P_2(\cos\theta) \right].
\]

(1)

d) The rotating sphere serves as an induction device if a stationary circuit is attached by a slip ring to the pole and a sliding contact at the equator. Show that the electromotive force in this circuit is \( E = \mu_0 m\omega/(4\pi R) \).