1. i) The condition to be satisfied is \( d \sin \theta = \lambda / 4 \). Therefore \( \sin \theta = 600 \text{mm} / (1 \text{mm}) = 6 \times 10^{-4} \). For such a small angle, \( \sin \theta \approx \theta \) and \( \theta = 6 \times 10^{-4} \) radians.
   
   ii) The screen extends 0.5 m on either side of the midplane of the two slits. Therefore \( \tan \theta \) ranges from \(-0.5 \text{m} / (2 \text{m})\) to \(0.5 \text{m} / 2 \text{m}\), i.e. \(-1/4 < \tan \theta < 1/4\). Therefore \( \sin \theta = \tan \theta / \sqrt{1 + \tan^2 \theta} \) lies between \(-0.242\) and \(0.242\). Therefore \( d \sin \theta / \lambda \) lies between \(-101.06\) and \(101.06\). There are \(101 \times 2 + 1 = 203\) bright fringes on the screen.

2. i) \( d \sin \theta = 2 \lambda \) when \( \tan \theta = 0.4 \text{km} / (1 \text{km}) = 0.4\), from which we can find \(\sin \theta = 0.3714\). Since \(d = 300 \text{m}\), we have \(d \sin \theta = 111.4 \text{m}\). Since this is equal to \(2 \lambda\), we have \(\lambda = 55.7 \text{m}\).

   ii) The next intensity minimum occurs when \(d \sin \theta = 2.5 \lambda\), i.e. when \(\sin \theta\) is \(2.5/2 = 1.25\) times what it was at the second maximum, i.e. \(\sin \theta = 0.4642\). Therefore \(\tan \theta = 0.5241\), i.e. \(y = 524 \text{m}\). She has to travel another 124 m.

3. The total electric field at a point is

\[
E(t) = E_0 [\sin(\omega t - 2\pi s_1 / \lambda) + \sin(\omega t - 2\pi s_2 / \lambda) + \sin(\omega t - 2\pi s_3 / \lambda)] \\
= 2E_0 \sin(\omega t - 2\pi s_2 / \lambda) \cos(\pi(s_3 - s_1) / \lambda) + E_0 \sin(\omega t - 2\pi s_2 / \lambda) \\
= E_0 [2 \cos(\pi(s_3 - s_1) / \lambda) + 1] \sin(\omega t - 2\pi s_2 / \lambda)
\]

(1)

where we have used the fact that \((s_1 + s_3)/2 = s_2\). When we calculate the intensity, the oscillatory part outside the brackets has a time average of half, and

\[
I(\theta) \propto [2 \cos(2\pi d \sin \theta / \lambda) + 1]^2
\]

(2)

The maxima can be found by requiring \(dI(\theta)/d\theta = 0\), which yields \(\sin(2\pi d \sin \theta / \lambda) = 0\). Therefore \(\cos(2\pi d \sin \theta / \lambda) = 1\) at the primary maxima, and \(\cos(2\pi d \sin \theta / \lambda) = -1\) at the secondary maxima. The ratio of the intensity at the secondary maxima to the intensity at the primary maxima is \([-2+1]^2/[2+1]^2 = 1/9\).

4. i) \(y(x)\) is continuous at \(x = 0\) because the two strings are knotted together, so they connect at \(x = 0\). The vertical component of the tension force (recall the derivation of the wave equation in a string) is \(F_T \partial y / \partial x\). If this is discontinuous at \(x = 0\) then, following the derivation of the wave equation in the string, there is a net vertical force exerted on the point of the ropes at \(x = 0\). Since the mass concentrated at any point is zero, a net vertical force would result in an infinite vertical acceleration, which is impossible.

   ii) Equating \(y(x < 0)\) and \(y(x > 0)\) at \(x = 0\), we have \(A \sin(-\omega t) + B \sin(\omega t) = C \sin(-\omega t)\), i.e. \(A - B = C\). Equating \(\partial y / \partial x\) on the two sides
of } x = 0 \text{, we have } (A\omega/v_1)\cos(-\omega t) + (B\omega/v_1)\cos(\omega t) = (C\omega/v_2)\cos(-\omega t), \\
\text{i.e. } (A + B)/v_1 = C/v_2. \text{ Combining these equations, we obtain}

\begin{align*}
B &= \frac{C}{2}(v_1/v_2 - 1) \\
A &= \frac{C}{2}(v_1/v_2 + 1).
\end{align*}

(3)

From these equations, it is easy to obtain that

\begin{align*}
C &= \frac{2A}{v_1/v_2 + 1} \\
B &= A\frac{v_1/v_2 - 1}{v_1/v_2 + 1}.
\end{align*}

(4)

When \( v_2/v_1 \to 0 \), i.e. \( v_1/v_2 \to \infty \), we have \( A/B \to 1 \). On the other hand, when \( v_1/v_2 \to 0 \), we have \( A/B \to -1 \).

5. The phase difference between the waves reflected from the bottom surface and the waves reflected from the top surface is \( \pi \) at one end of the plates and \( \pi + (2 \times 0.06mm/500nm) \times 2\pi = \pi(1 + 480) \) at the other end. Bright bands occur when the phase difference between the two is equal to \( 2\pi, 4\pi, 6\pi \ldots \). Therefore there are 240 bright bands.

6. Because the refractive index of the metal oxide is greater than that of glass, the top surface of the layer has a medium of lower refractive index above it and a medium of higher refractive index below it, while the bottom surface has a medium of higher refractive index above it and a medium of lower refractive index below it. Therefore the phase difference between the waves reflected from the two surfaces is \( \pi + 2\pi(2d/\lambda) \). Destructive index occurs when \( 2d \) is an integer multiple of \( \lambda/2.61 = 550nm/2.61 = 210.73nm \), the wavelength of the light inside the metal oxide. Therefore the smallest possible thickness is \( 210.73nm/2 = 105.4nm \) while the next possible thickness is \( 210.73nm \).